# Generation and Transport of Vorticity and Effects on Mean Surface Currents: Wave Averaged and Wave Resolving Formulations

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## LONG-TERM GOALS

Our long term goal is to understand the mechanisms controlling the intensity and spatial distribution of vortical structures, such as eddies and rip currents, evolving in the surfzone, and to understand how these flow features influence time-averaged currents, sediment transport and wave statistics.

## OBJECTIVE

Recent computations (Kirby et al, 2003a, b) have shown that computations of shear waves using either wave-averaged circulation models on the one hand, or wave-resolving Boussinesq models on the other hand, can predict very different nearshore circulation patterns when configured similarly and applied to the same field cases. This discrepancy is troubling, in that a clear basis has not been established for determining which results are typically closer to observed field conditions. In order to resolve this issue, we plan to perform comparisons of model runs for a number of time periods from the SandyDuck field experiment, and compare results to both array measurements of long-shore and cross-shore velocities as well as Doppler Sonar measurements of the study area, which provide more information on the spatial structure of flow features.

Specific objectives in support of this effort include:

1. Re-examine the energetics and spatial structure of low-frequency vortical motions as predicted by the Boussinesq model FUNWAVE and the wave-averaged circulation model SHORECIRC. The comparison will be performed using a newly-revised Boussinesq formulation which preserves an approximation for potential vorticity which is consistent with Boussinesq ordering in powers of .

2. Test model predictions against available array data and Doppler sonar data.

3. Examine the instantaneous structure of forcing for vorticity derived from Boussinesq simulations, based on the curl of momentum-preserving dissipative terms.

4. Examine the time-averaging of this forcing in the context of forcing of wave-averaged vorticity.

5. Examine the contributions to wave-averaged radiation stress derived from Boussinesq model results, and examine the curl of this stress and its relation to the time average of instantaneous vorticity forcing.

6. Formulate the generalized Lagrangian mean (GLM) equations for Boussinesq model flows, and examine the relative importance of contributions to GLM forcing terms based on direct examination of time-resolved Boussinesq model calculations. In addition to formulating the GLM averaged equations,

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we will also perform Lagrangian particle trajectory calculations in the wave-resolving Boussinesq code in an attempt to verify the GLM predictions and evaluate the importance of resulting terms.

These objectives are aimed at obtaining a better understanding of the marked differences in predictions of wave-averaged circulation models and Boussinesq models for similar field conditions, as outlined below.

## WORK COMPLETED

Work completed in the second year of the project was concentrated in three areas: (1) An analysis of the relative importance of terms leading to vorticity production and decay in wave resolving and wave-averaged models, (2) a comparison of radiation stress and CL-vortex force formulations in 2-D and quasi-3D wave-averaged circulation models, and (3) an analysis of nearshore dispersion predicted by wave-resolving and wave-averaged models based on numerically simulated Lagrangian particle pair statistics.

*Vorticity production and decay in wave resolving and wave-averaged models*

An analysis of the mechanism for the generation and decay of vertical vorticity in the wave-resolving Boussinesq model of Wei et al (1995) and the 2-D, depth-integrated Shorecirc module of NearCoM has been described by Terrile et al (2006). This work is continuing in the context of a cooperative project between Kirby, Brocchini and Briganti, with Emanuele Terrile (U. Genova) and Joe Geiman (U. Del.) as graduate students. Presently, we are performing an analysis of wave-averaged motions in the Boussinesq model in a Generalized Lagrangian Mean (GLM) formulation.

*Radiation Stress vs. CL-vortex force formulations in 2D and quasi-3D circulation models*

We have completed an analysis of the correspondence between the forcing mechanisms in the 2-D, depth-averaged Shorecirc module of the NearCoM model, and have shown their equivalence when full feedback is maintained between Shorecirc and the wave driver. These results, and preliminary analysis of the quasi-3D formulation, have been described by Shi et al (2006a,b) and will be discussed more fully in a manuscript in preparation.

*Nearshore dispersion from Lagrangian particle trajectories.*

An analysis of nearshore dispersion estimated from 1 and 2 point statistics has been carried out for an example shear wave case, using numerically simulated Lagrangian particle trajectories. Resuolts are described below. We are presently investigating various strategies for seeding the Eulerian flow field, in preparation for performing simulations using large numbers of Lagrangian markers.

## RESULTS

Results of the study of vorticity generating mechanisms in Boussinesq (wave-resolving) and wave-averaged models have led to a more complete picture of the mechanisms in each case, but have not shed much light yet on fine details in differences between the formulations. The initial analysis comparing the Boussinesq model of Wei et al (1995) to the wave-averaged 2-D version of Shorecirc are presented in Terrile et al (2006). In parallel to this analysis, a more complete analysis of the differences between the Boussinesq model formulations of Wei et al (1995), Chen et al (2003) and Chen (2006) has been performed. In particular, the formulation of Chen (2006) has been shown to

preserve the usual measure of potential vorticity to the order of terms retained in a weakly dispersive Boussineqs model. This property is not preserved by the models of Wei et al (1995) and Chen et al (2003), prompting the question of whether there are important differences in the dynamics predicted by the different formulations. Gobbi et al (2006) began an analysis of this question by examining the vorticity and global enstrophy predictions during simulations of a topographically controlled rip current (Chen et al, 1999) and an energetic shear wave condition (Kirby et al, 2003a,b). Simulations of shear waves over longshore-uniform barred bathymetry (Figure 1) using the three models show essentially no differences in predictions of the Eulerian motion field as diagnosed by vorticity and global enstrophy – for example, Figure 2 shows the growth of total enstrophy integrated over the domain for a given forcing condition in the three models..

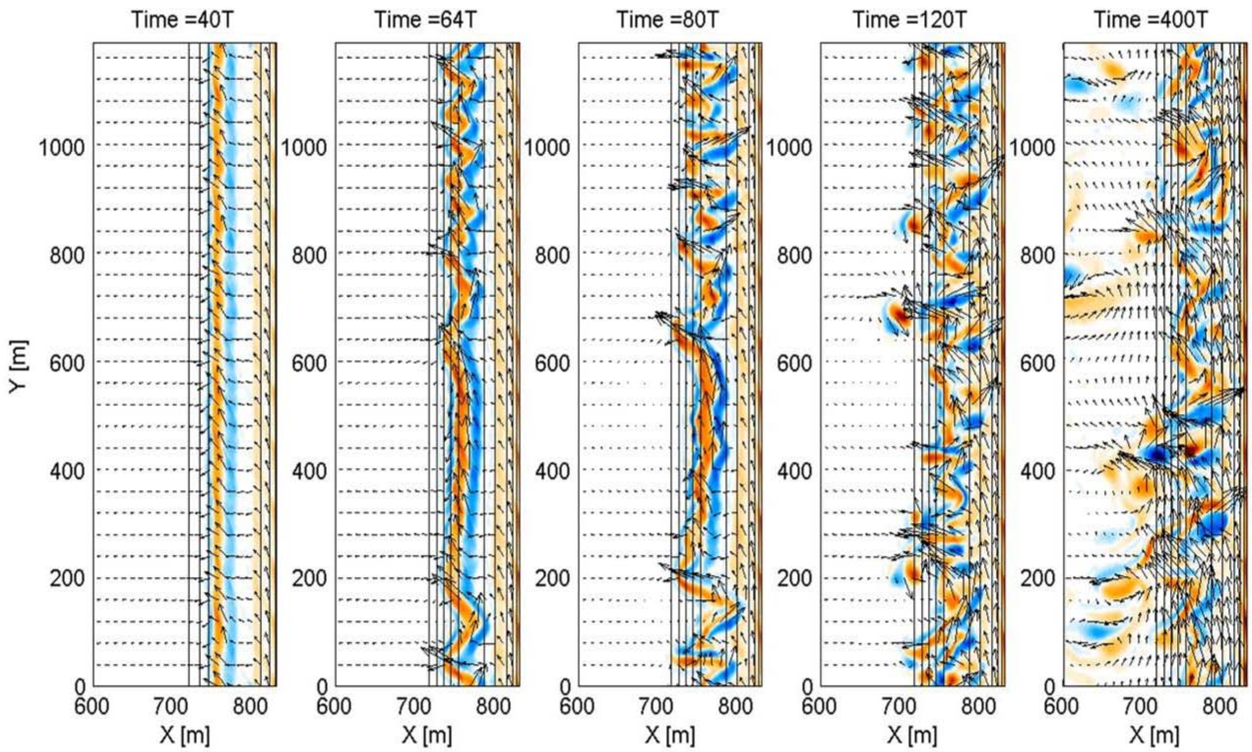


***Figure 1: Growth of a shear wave instability and subsequent evolution to a complex eddy field for an alongshore current on an alongshore-uniform barred beach. Model of Chen (2006).***

In pursuit of a deeper understanding of the models, we are presently examining one- and two-point dispersion statistics based on simulations of Lagrangian particle trajectories. Our approach is to perform and store the velocity field for an entire run of the Eulerian model. The stored velocity field is then used to specify velocities at arbitrary locations and times by interpolation within the stored matrix of Eulerian velocities, in order to integrate Lagrangian particle trajectories forward in time. Then, using methods described in, for example, LaCasce and Bower (2001), we have examined absolute and relative longshore and cross-shore diffusivities in longshore current systems. Figure 3 shows a plot of total relative diffusivity (top row) and longshore diffusivity (bottom row) for (from left to right), the Boussinesq model of Chen et al (2003), the Boussinesq model of Chen (2006), and the wave-averaged model Shorecirc for the case of an unstable shear wave motion. All models indicate an ultimate approach to a D4/3 behavior, indicating the likely dominance of shear in the total and longshore

diffusivity. This result is in agreement with field measurements reported by Johnson and Pattiaratchi (2004). We note that all the models approach this behavior for cross-shore diffusivity as well, but that the result does not continue to hold at long times, where there is an indication that cross-shore diffusivity is limited by the characteristic lengthscale representing the shoreline to bar crest distance. Preliminary results have been presented by Briganti et al (2006) and will be amplified by Kirby et al (2006).

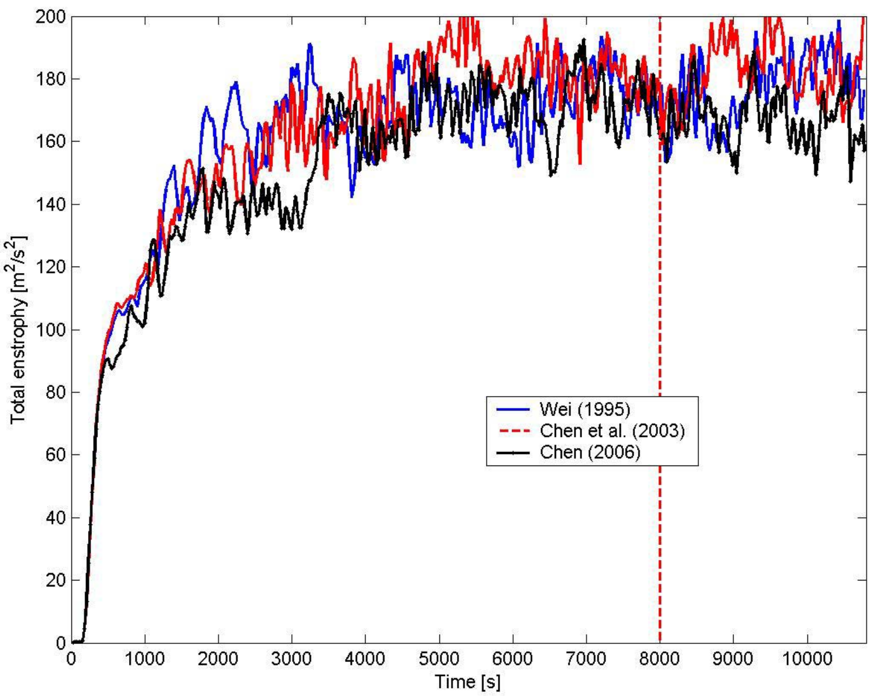


***Figure 2: Growth of global enstrophy during the spinup of the instability of the longshore current during simulations based on three versions of the fully nonlinear, weakly dispersive Boussinesq model. There is no distinct difference between the three models evidenced by the global enstrophy balance for this case.***

Shi et al (2006a,b) have presented an analysis of the correspondence between radiation stress and CL-vortex force formulations in 2-D (depth averaged) and quasi-3D wave-averaged formulations. For cases where full wave-current effects are retained in both the 2D circulation and wave driver models, Shi et al (2006a) showed that the two formulations are equivalent. Perhaps most importantly, the results for cases where wave current feedback is not retained (as would be the case when a circulation model is driven by a previously calculated wave field which is not in turn updated) show that the use of a constant forcing field in the CL-vortex formulation produces a more accurate circulation pattern than does use of a constant forcing based on the radiation stress field. This indicates that forcing in the uncoupled case should be calculated in the CL-vortex form as the routine approach. These results, and an analysis of the quasi-3D results, will be presented shortly in a manuscript for publication.

**IMPACT/APPLICATIONS**

Transport and mixing of materials near shore has broad implications, from dispersal of pollutants to fish recruitment, from bottom morphology to sediment transport. The action of vortical motions is

central to this. Understanding how wave forcing affects the motions, including this “turbulence,” is central to understanding the system.

Wave-current interactions are a significant influence near shore, and (indeed) in the oceans in general. Improved understanding, and validation of the results, has the broadest possible implications

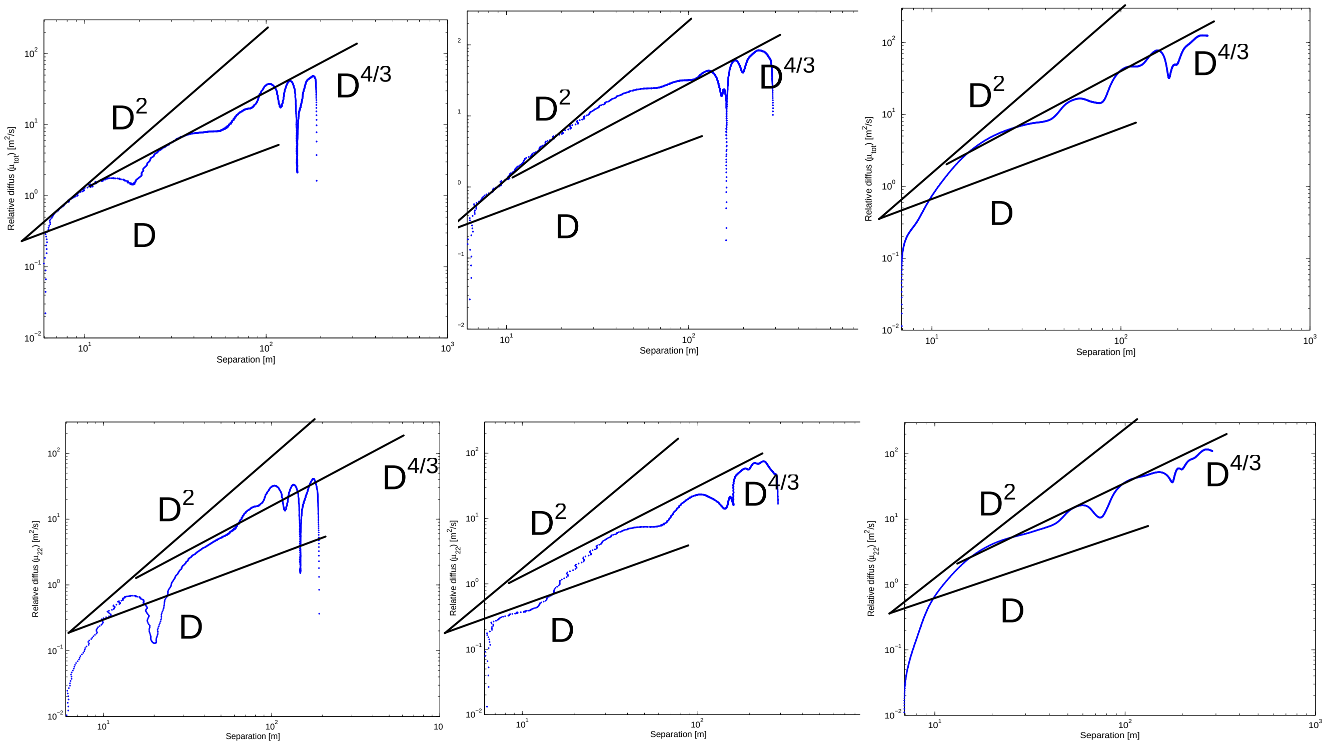


***Figure 3: Total relative diffusivity (top row) and alongshore diffusivity (bottom row) for (from left to right) the Boussinesq model of Chen et al (2003), the Boussinesq model of Chen (2006), and Shorecirc.***

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